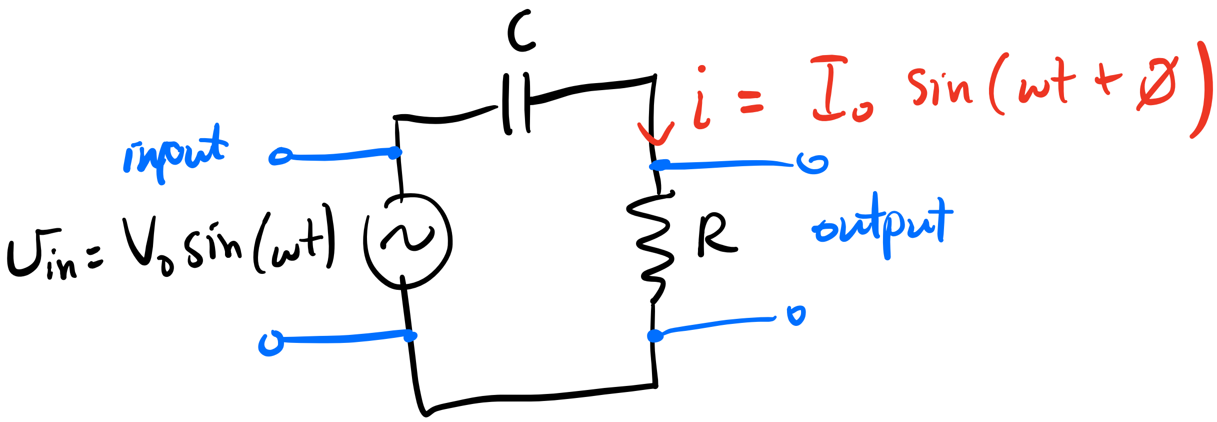
PHYS 231 - Oct. 11, 2023

No Lab this week. (Oct. 12, 2023)

Last Time:

RC circuit frequency response



Goal was to find $I_o(\omega)$ & $\phi(\omega)$

$$\Rightarrow \quad \tan\phi = \frac{1}{\omega RC}$$

$$I_o = \frac{\omega V_o C}{\sqrt{1 + (\omega RC)^2}}$$

① Practical use of the RC circuit driven by sine wave.

Consider voltage across resistor

$$V_R = iR = \underbrace{I_0 R}_{\sim} \sin(\omega t + \phi)$$

$V_R = I_0 R$: Amplitude of voltage across resistor.

Know $I_0(\omega)$, therefore

$$\boxed{V_R = \frac{\omega V_0 RC}{\sqrt{1+(\omega RC)^2}}}$$

amplitude of signal generator output.

Consider ratio of Resistor amplitude to fcn generator amplitude.

$$\frac{V_R}{V_0} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$$

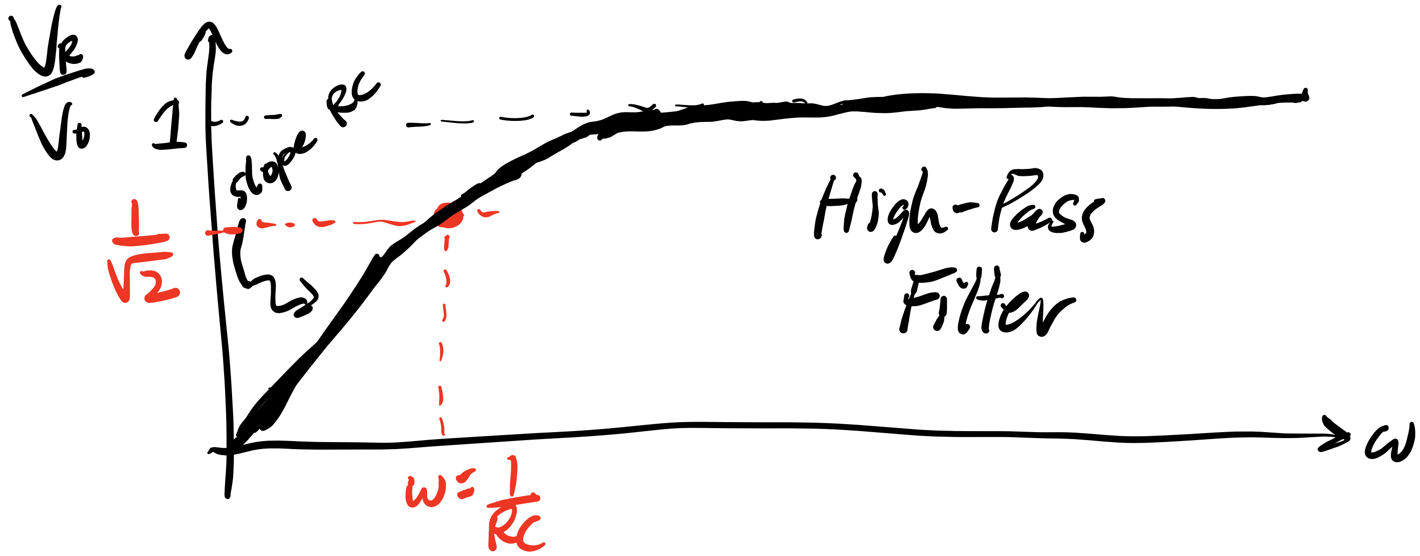
Plot V_R/V_0 vs ω

limit $\omega \to 0$

$$\frac{V_R}{V_0} \approx \omega RC$$

limit $\omega \to \infty$

$$V_R/V_0 \approx \frac{\omega RC}{\omega RC} = 1$$



Consider the angular freq. $\omega = \frac{1}{RC}$

$$\frac{V_R}{V_0} = \frac{\left(\frac{1}{\cancel{RC}}\right)\cancel{RC}}{\sqrt{1+\left(\frac{1}{\cancel{RC}}\cancel{RC}\right)^2}} = \frac{1}{\sqrt{2}} = 0.707$$

If we take V_R as the output of our circuit:

- At high freq., the input (provided by fcn gen.) is passed to the output;

- At low freq., the output is $\underbrace{\text{attenuated}}_{\text{suppress amp. of output.}}$ compared to the input

This circuit acts as a "High-Pass" filter since it passes signals at high freq. & attenuates signals at low freq.

---

(2) Develop Complex Numbers as a mathematical to help us analyze circuits in the frequency domain.

<u>Definition:</u> $\sqrt{-1} = \pm j$

$$\Rightarrow -1 = (\pm j)^2 = (\pm 1)^2 (j)^2$$

$$\therefore j^2 = -1$$

Why is this definition useful?

- It allows to solve some problems that previously had no obvious sol'n.

Eg. Consider $\sqrt{-36} = \underbrace{\sqrt{-1}}_{\pm j}\,\underbrace{\sqrt{36}}_{6}$

$$\Rightarrow \quad \sqrt{-36} = \pm 6j$$

Eg. Consider $\overset{a}{1}x^2 \overset{b}{-2}x + \overset{c}{4} = 0$

→ find sol'ns for x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-12}}{2} = 1 \pm \frac{\overset{j}{\sqrt{-1}}\sqrt{12}}{2}$$

$$x = 1 \pm j\,\frac{\cancel{\sqrt{4}}\sqrt{3}}{\cancel{2}}$$

$$\boxed{x = 1 \pm j\sqrt{3}}$$

We say that x has two components:
"Real" part & "Imaginary" part

Definitions:

$$\underbrace{Re[x]}_{\text{"real part of } x\text{"}} = 1$$

anything not multiplied by j

$$\underbrace{Im[x]}_{\text{"imaginary part of } x\text{"}} = \pm\sqrt{3}$$

the stuff multiplied by j.

(does not include the factor of j).

Check that our sol'n for x works

sub $x = 1 \pm j\sqrt{3}$ into

$$x^2 - 2x + 4 = 0.$$

$$x^2 = (1 \pm j\sqrt{3})^2 = (1 \pm j\sqrt{3})(1 \pm j\sqrt{3})$$

$$= 1 \pm j\sqrt{3} \pm j\sqrt{3} + (\pm j\sqrt{3})^2$$

$$= 1 \pm 2j\sqrt{3} + \underbrace{j^2}_{-1} 3$$

$$= \underline{-2 \pm 2j\sqrt{3}}$$

$$\therefore x^2 - 2x + 4 = (-2 \pm 2j\sqrt{3}) - 2(1 \pm j\sqrt{3}) + 4$$

$$= \cancel{-2} \cancel{\pm 2j\sqrt{3}} \cancel{-2} \cancel{\mp 2j\sqrt{3}} + \cancel{4}$$

$= 0$ as expected.

In general, any complex no. can be expressed in the form:

$$z = \underset{\uparrow}{x} + j\underbrace{y}$$

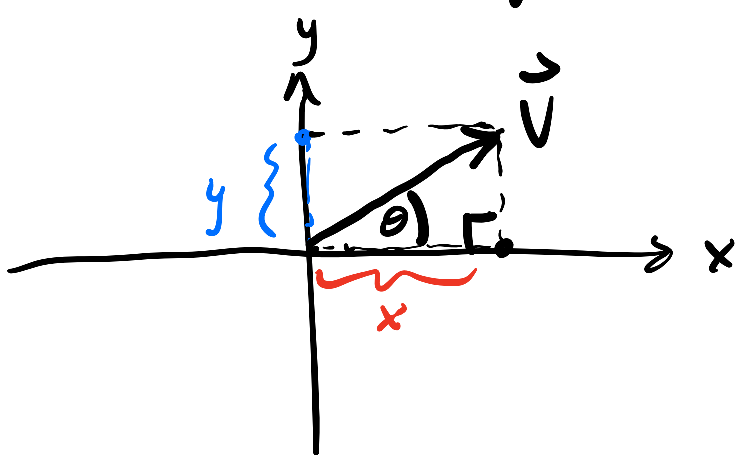
real component

imaginary component

The expression $z = x + jy$ resembles a two-component vector $\vec{V} = x\hat{i} + y\hat{j}$

$\vec{v} = x\hat{i} + y\hat{j}$	x-component x	y-component y
$z = x + jy$	real component x	img. component y

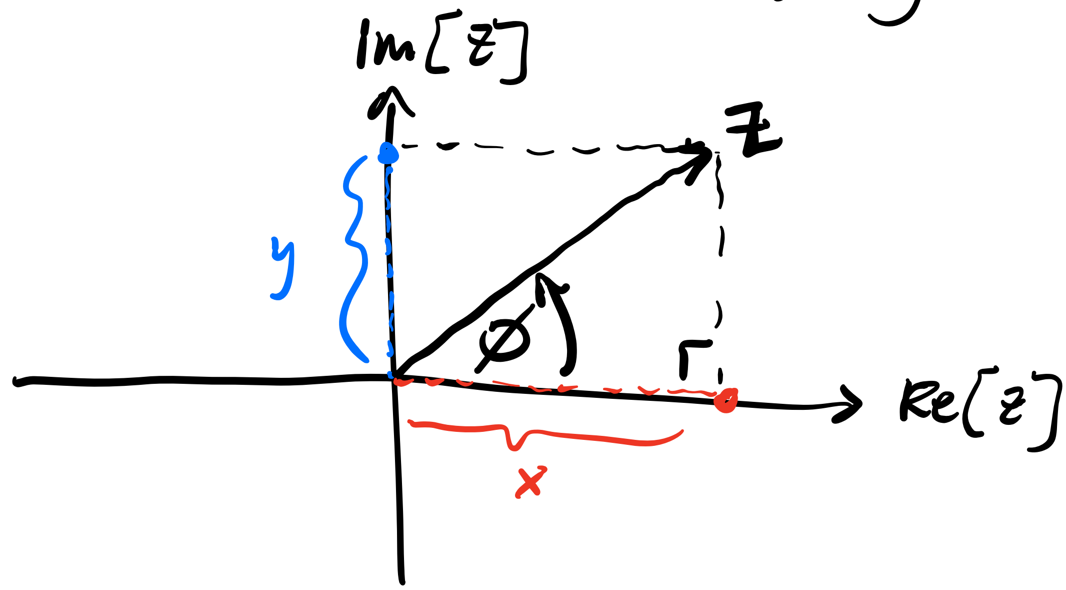
If we plot $\vec{v}$ in x-y plane



$$|\vec{v}| = \sqrt{x^2+y^2}$$

$$\tan\theta = \frac{y}{x}$$

By analogy, let's plot complex number $z = x + jy$ in the so-called complex plane which has a real axis & an imaginary axis



complex no. z has a magnitude or "length"

$$|z| = \sqrt{x^2 + y^2}$$

$$= \sqrt{Re[z]^2 + Im[z]^2}$$

$$\tan\phi = \frac{y}{x} = \frac{Im[z]}{Re[z]}$$